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Note

A Note on Larson's Particle Method

Larson (1978) has suggested a method for numerically simulating an isothermal, self gravitating gas using interacting particles. The algorithm is based on a physical argument which assumes each particle represents a spherical bag of gas, and leads to two such particles repelling each other with an acceleration which is along their line of centres and varies inversely with their separation. In Larson's work the total force on a given particle is due to its nearest neighbours. In this note I shall show how Larson's algorithm can be obtained from first principles and indicate how it can be improved and generalized.

Let the acceleration of particle i at $\mathbf{r}_i$ due to its nearest neighbours be

$$am \sum_j \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^2}, \tag{1}$$

where m is the mass of a particle, a is a constant which we specify later, and the summation is over nearest neighbours. We define a nearest neighbour of particle i in three dimensions such that

$$|\mathbf{r}_i - \mathbf{r}_j| \leqslant \sigma/\rho(\mathbf{r}_i)^{1/3}, \tag{2}$$

where σ is a constant chosen so that the number of nearest neighbours is sufficiently large to allow (1) to be replaced by the integral

$$a \int \frac{(\mathbf{r}_i - \mathbf{r}')}{|\mathbf{r}_i - \mathbf{r}'|^2} \rho(\mathbf{r}')\, d\mathbf{r}'. \tag{3}$$

The summation could be regarded as a Monte Carlo estimate of (3).

If the variation of ρ within the nearest neighbour sphere is sufficiently small, (3) can be approximated by

$$a \int \frac{(\mathbf{r}_i - \mathbf{r}')}{|\mathbf{r}_i - \mathbf{r}'|^2} \{\rho(\mathbf{r}_i) + (\mathbf{r}' - \mathbf{r}_i) \cdot \nabla\rho(\mathbf{r}_i)\}\, d\mathbf{r}';\ |\mathbf{r}' - \mathbf{r}_i| < \sigma/\rho(\mathbf{r}_i)^{1/3}, \tag{4}$$

which reduces to

$$-\frac{4\pi a}{3} \sigma^3 \frac{\nabla\rho}{\rho}. \tag{5}$$

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The choice

$$a=\frac{3}{4\pi\sigma^3}\frac{RoT}{\mu}, \tag{6}$$

leads to

$$\frac{d^2\boldsymbol{r}}{dt^2}=-\frac{\nabla P}{\rho} \quad \text{with} \quad P=\frac{RoT\rho}{\mu}. \tag{7}$$

The motion of an isothermal gas is therefore simulated by Larson's algorithm provided there are enough nearest neighbours to ensure (1) is a good approximation to (3). Larson only uses an average of about 1.5 nearest neighbours and it would appear from the above analysis that very large spurious fluctuations will occur.

For a barytropic gas (1) can be easily generalized to give

$$\frac{d^2\mathbf{r}_i}{dt^2}=\frac{3m}{4\pi\sigma^3}\sum_j\left(\frac{P}{\rho}\right)_j\frac{\mathbf{r}_i-\mathbf{r}_j}{|\mathbf{r}_i-\mathbf{r}_j|^2}. \tag{8}$$

However, this algorithm does not exactly conserve either linear or angular momentum. A conservative algorithm results if instead we consider

$$\frac{d^2\mathbf{r}_i}{dt^2}=\frac{3m}{4\pi}\sum_j\left(\frac{P_i}{\rho_i^2}+\frac{P_j}{\rho_j^2}\right)\frac{\mathbf{r}_i-\mathbf{r}_j}{|\mathbf{r}_i-\mathbf{r}_j|^2}\frac{1}{H_{ij}^3}, \tag{9}$$

where, in the summation, we require

$$|\mathbf{r}_i-\mathbf{r}_j|<H_{ij}$$

with H_{ij} symmetrical in i and j. For example,

$$H_{ij}=\frac{\sigma}{[\rho(\mathbf{r}_i)\,\rho(\mathbf{r}_j)]^{1/6}}. \tag{10}$$

If the analysis leading from (1) to (7) is followed and variations of H_{ij} are neglected, (9) will be found to reduce to

$$\frac{d^2\mathbf{r}}{dt^2}=-\left[\frac{P}{\rho^2}\nabla\rho+\nabla\left(\frac{P}{\rho}\right)\right]=-\frac{\nabla P}{\rho}. \tag{11}$$

The resulting algorithm conserves linear and angular momentum exactly and has the nice feature of a resolution (αH_{ij}) which varies in space and time.

It is interesting to note that Larson's algorithm is really a kernel estimate (Gingold and Monaghan 1978, 1981) with the kernel

$$W(\mathbf{u}, h) = -\frac{q}{h^3 4\pi} \ln(u/h); \qquad 0 \leqslant u \leqslant h,$$
$$= 0; \qquad u > h, \tag{12}$$

where the smoothing length h is H_{ij}. The algorithm is therefore a special case of smoothed particle hydrodynamics (SPH).

References

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